First arrival tomography using depth-varying velocity gradients

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Summary

Many cell and layer tomography methods assume that the velocity or velocity perturbation is constant within each cell or layer. When the velocity field varies continuously, traditional cell or layer tomography methods will be problematic. While it will be erroneous to approximate the velocity field using few constant-velocity cells, using two many cells will increase the number of inversion variables and the computation cost. Here we improve a first arrival layer tomography to invert for the interface geometry and depth-varying velocity gradient of each model layer. An efficient ray tracing is developed for the model with depth-varying velocity gradients. For areas of prominent velocity gradient, our new method will use fewer model parameters and therefore more robust tomographic inversion than the method using constant velocity cells or layers. The method is shown using a 2D synthetic example.

Introduction

Tomography has become an effective way of velocity model building for static corrections, migration velocity analysis, and lithologic interpretation. One common method is cell tomography, which models the velocity field using a number of cells, and inverts for the velocities of these cells. Another method is layer tomography that is applicable where geologic features such as weathering zones, stratigraphic units, and salt bodies can be represented easily by layers. Layer tomography may directly invert for the geometry of layer interfaces. Many cell and layer tomography methods assume that the velocity or velocity perturbation is constant within each cell or layer.

However, velocity field often changes continuously with depth, such as the case for young sediments due to compaction. Among different relationship between velocity and depth, velocity increasing linearly with depth is of considerable practical importance. Within the top several kilometers of the Earth, the vertical velocity gradient is generally between 0.3/s and 1.3/s (Sheriff and Geldart, 1995). A typical velocity gradient in the Gulf of Mexico is 0.5/s (Biondi, 2006). Large velocity gradients are likely present within near-surface materials and near the water table in unconsolidated material (Birkelo et al., 1987; Miller and Xia, 1997, 1998). Ettrich (2002) shows that models with constant velocity gradient approximate the real earth much better than models with constant velocities.

If the subsurface has prominent velocity gradients, using constant velocities for tomographic inversion is challenging, especially using fewer cells or layers. For example, in the case of inverting for layer interfaces, if slower interval velocities are used, the layer interfaces will be shallower in the solution model. If faster velocities are used, the interfaces will be deeper. Some decades ago, people considerably underestimated the depth to the Moho interface between crust and mantle because of using velocities got directly from first arrival curves. The reason is because the seismic velocity in oceanic crust increases relatively smoothly with depth as a series of gradients rather than in two or three uniform velocity layers. The first arrivals used come mainly from rays which have turned in the upper part of the velocity gradient, because rays from deeper in the layer are masked as second arrivals. Since this apparent velocity is assumed to apply as a uniform value throughout the layer in inversion, adoption of too low an overall velocity results in calculation of too thin a layer (White, 1992). So using constant-velocity cells or layers to model subsurface of real earth might be erroneous.

People may use more constant-velocity cells or layers to model the subsurface with prominent velocity gradients, however, this will involve more model parameters, which means more computation expense for ray tracing and inversion will not be robust any more.

Geologic activities, salt intrusion, and others made the layers with velocity gradients deformed, such as anticlines. The velocity gradients are still kept in each layer. So we call these depth-varying velocity gradients. We developed a technique which can model subsurface having depth-varying velocity gradients and the solutions of tomographic inversion are more stable and accurate without expensive computation.

Method

Good model parameterization for inversion must use a minimum number of model variables, be simple to implement, and be efficient for computation (Zhou, 2006). The models in our method consist of a number of layers based on a stratigraphic interpretation. Figure 1 shows the model cell. The velocities are recorded at the four corners. In each cell, we allow a linear velocity gradient in it. Because it is not unique to get a gradient for four velocity values, we make an approximate. The two red solid dots are the middle points of the top and bottom boundary of the cell. Then the average velocity of the velocities at the top two corners is denoted by V_T. The average velocity of the velocities at the bottom two corners is denoted by **V_B**. The velocity gradient in this cell is defined as **a** = (**V_B**-**V_T**)/**h**.

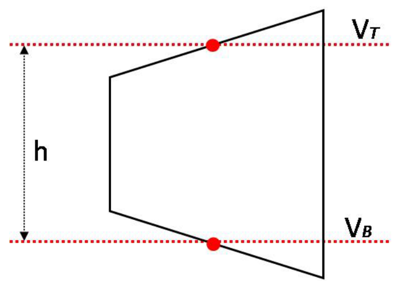

Figure 1: A model cell used in our method. Black trapezoidal is the cell. The two red dashed lines is an approximation to get the surface velocity and bottom velocity.

In our tomographic inversion, the objectives we want to invert for are the geometry of layer interfaces and/or velocity gradients for each layer. Equation (1) shows the relationship between traveltime residual, Frechet kernels, and model parameters.

$$\delta t_i = \sum_{j}^{J} S_{ij}\delta s_j + \sum_{k}^{K} A_{ik}\delta a_k + \sum_{l}^{L} Z_{il}\delta z_l \quad (1)$$

where, δt_i is the traveltime residual in i-th ray; S_{ij} is the slowness kernel of i-th ray and j-th cell; δs_j is the slowness perturbation of the j-th cell; A_{ik} is the velocity gradient kernel of i-th ray and k-th cell; δa_k is the gradient perturbation of the k-th cell; Z_{il} is the interface kernel of i-th ray and l-th grid point; δz_l is the interface perturbation of l-th grid point.

Inverting all the parameters simultaneously will bring ambiguity. A common way is to invert them progressively, or even invert only one of them based on other *a priori* information. The good thing is that we sometimes can get some information of layer velocities from well log and other geophysical or geologic surveys. That means in this case we don't need to invert for velocity and gradients and only need to invert for the geometry of layer interfaces. If we don't have well log data or any other help, we can use the technique introduced by Clayton and McMechan (1981), who used slant stack and downward continuation to get 1D velocity-depth profiles from refraction data without picking first arrivals. This kind of 1D velocity profile also can be used as velocity information in initial model replacing well log data.

Other details of methods will be shown in the example below.

A 2D synthetic example

In this example, a 2D first arrival traveltime data is got by ray tracing in a model with a series of velocity gradients and then adding Gaussian noise, whose standard deviation is 15 milliseconds. Our method of tomographic inversion is used to invert for this true geologic model from an initial model. Figure 2b is the true model with 3500 m long and 1000 m deep. The 2D survey used 23 sources (pink circles) and 26 receivers (blue triangles). The maximum offset is 3500 m. Red curves are first arrival ray paths. Interfaces between different layers are shown by black curves. The color bar shows the velocity values of the model. Figure 2a shows a 1D velocity profile at the position denoted by the black arrow in Figure 2b. This profile shows that there are three layers with a series of velocity gradients.

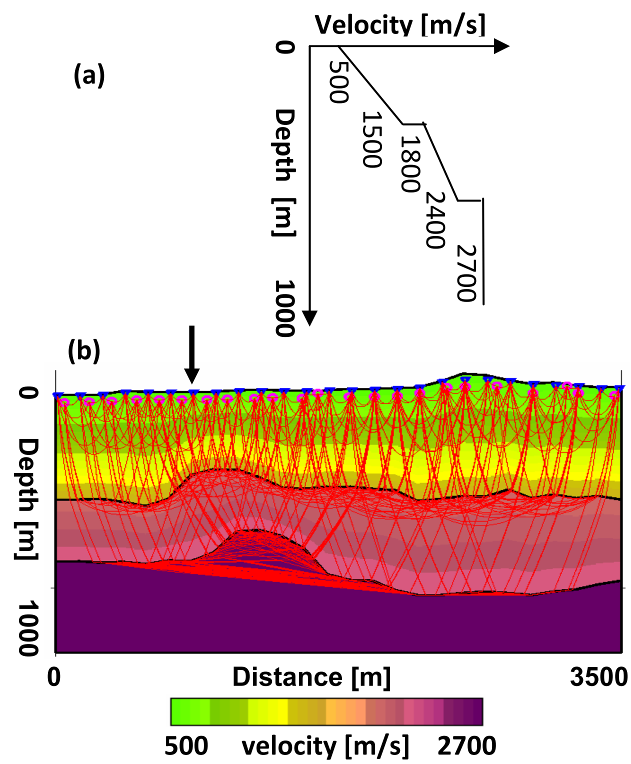

Figure 2: (a) 1D velocity profile at a position denoted by the black arrow. (b) A 2D velocity model which is 3500 m wide and 1000 m deep. This model will be regarded as the true model in the synthetic test.

This example composes of three different tests. Firstly, a traditional first arrival cell tomography result is shown. Secondly, our method is used to invert for the true model. We first use interval velocities picked from first arrival curves to invert for geometry of layer interfaces and then update the velocity gradients to get the final solution. In the third test, we assumed that the velocity information including its gradients is known from *a priori* information, such as from well log data, and we use this to invert for the geometry of layer interfaces.

Test 1:

This test uses traditional first arrival tomography. Because the subsurface in true model (Figure 2b) has prominent velocity gradients, more layers or cells should be used to model. Figure 3a is the initial model. The initial velocity values are chosen by fitting the first arrival curves. Figure 3b is the solution model after 10 iterations. In this test, no regularization is used in the inversion process. We just illustrate the well known truth that, due to the large number of model parameters, the inversion result is not robust compared to the true model in Figure 2b. Besides, the computation is expensive both in ray tracing and inversion also due to the large number of model parameters.

Test 2:

This test shows a practical tomographic inversion procedure of our method. Firstly, we determine the interval velocities from first arrival curves and use these interval velocities to invert for geometry of layer interfaces. Different with the way of drawing some straight lines to fit the curves and then using the slope as slowness, we draw the straight lines going through the prominent turning points in the curve (red lines are drawn crossing these points in Figure 4a). These turning points are related to those layer interfaces where a prominent jump of velocities often exists. Drawing straight lines through turning points can lower the risk of picking slower or faster velocities. The slopes of these straight lines are considered as inverse of interval velocities. These interval velocities are used in initial model (Figure 4b). Secondly, use these interval velocities to invert for geometry of layer interfaces, the solution of this stage is shown in Figure 4c. Finally, we use the model in Figure 4c as initial model to invert for velocity gradients, which also can be called gradients updating. Figure 4d is the final solution. Compared to true model in Figure 2b, this is much better than the solution (Figure 3b) got by traditional first arrival cell tomography. We invert for geometry first and update velocity gradients later because geometry and gradient kernels in Equation 1 have different sensitivities.

Test 3:

It is common that, besides seismic data, we also can get well log or other geophysical/geological survey data to constrain our tomographic inversion. In other words, if we have a well log which shows the general trend of velocity variations, they can be used directly for our initial model, and then the main inversion objects will be the geometry of layer interfaces. This test is based on this assumption. Figure 5a is the initial model. Figure 5b is the solution model after 10 iterations. Compared to true model in Figure 2b, this solution is slightly better than solution model in Figure 4d, because of using *a priori* information to constrain. This is also an advantage of our method. Given a well log profile with a series of prominent velocity gradients, traditional methods cannot use them directly but just using many constant velocities to approximate. However, using many model parameters means more computation expense and less robust inversion results.

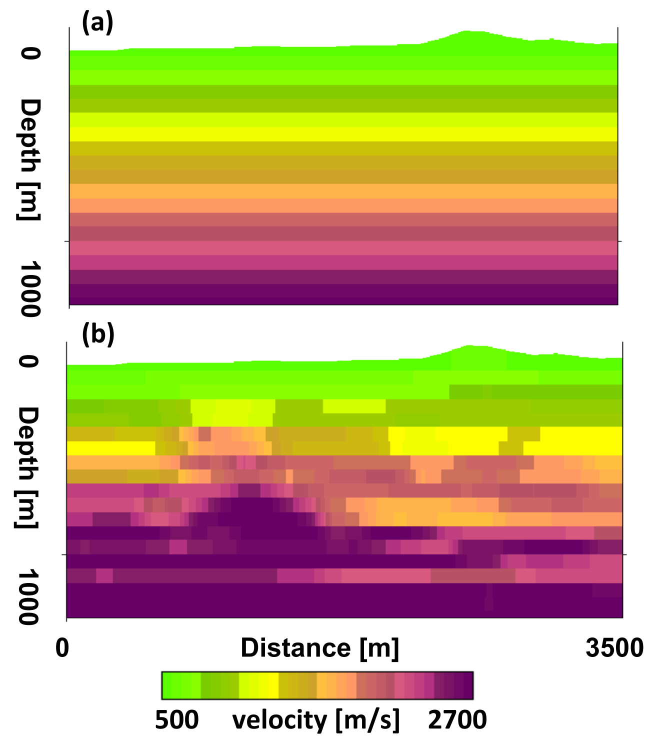

Figure 3: Test of a traditional first arrival cell tomography. (a) The initial model. (b) Solution after 10 inversion iterations without regularization.

Discussion and conclusions

A new method of first arrival layer tomography is devised to incorporate vertical velocity gradient. At areas with prominent velocity gradients, the new method uses fewer model parameters and therefore more robust tomographic inversion in comparison with previous tomographic methods using constant-velocity cells or layers. A synthetic test shows that, comparing with the traditional cell tomography, the new gradient method can yield more accurate and stable solutions with lower computation cost.

It is common that other information such as well log data can provide us velocity information, we only need to invert for geometry of layer interfaces in this case. If well log data shows prominent velocity gradients, our method can use the information directly, avoid any approximation using constant velocities.

Since the method is an improved layer tomography, it still has limitations. Layer tomography is applicable where geologic features such as weathering zones, stratigraphic units, and salt bodies can be represented easily by layers. In cases of very little or no velocity gradient, we may still use the traditional tomography methods with constant-velocity cells or layers.

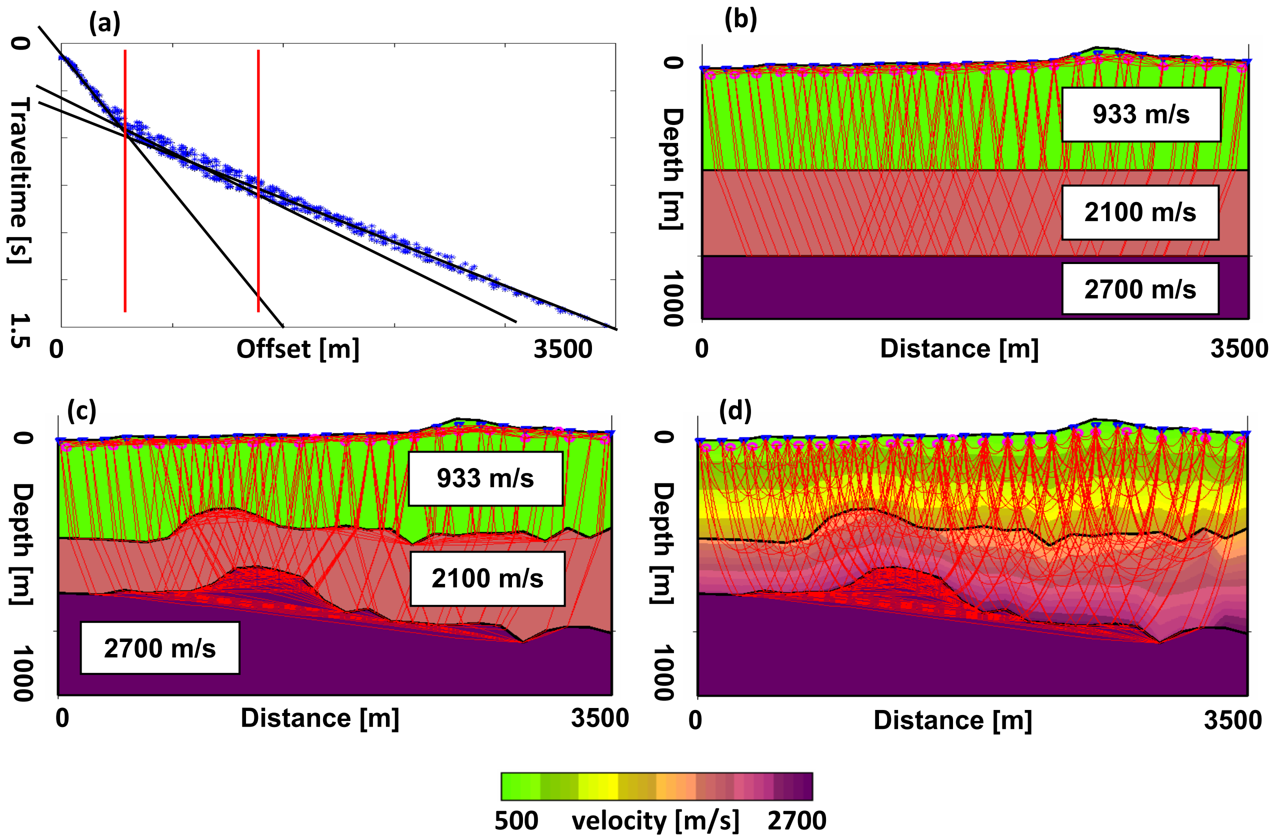

Figure 4: First arrival tomography using depth-varying velocity gradients. (a) The first arrival data from which the interval velocities are picked. (b) The initial model using constant interval velocities estimated from (a). (c) Solution model from inversion for interface geometry of the constant layer velocity model. (d) Solution model after updating velocity gradients using model in (c) as the initial model.

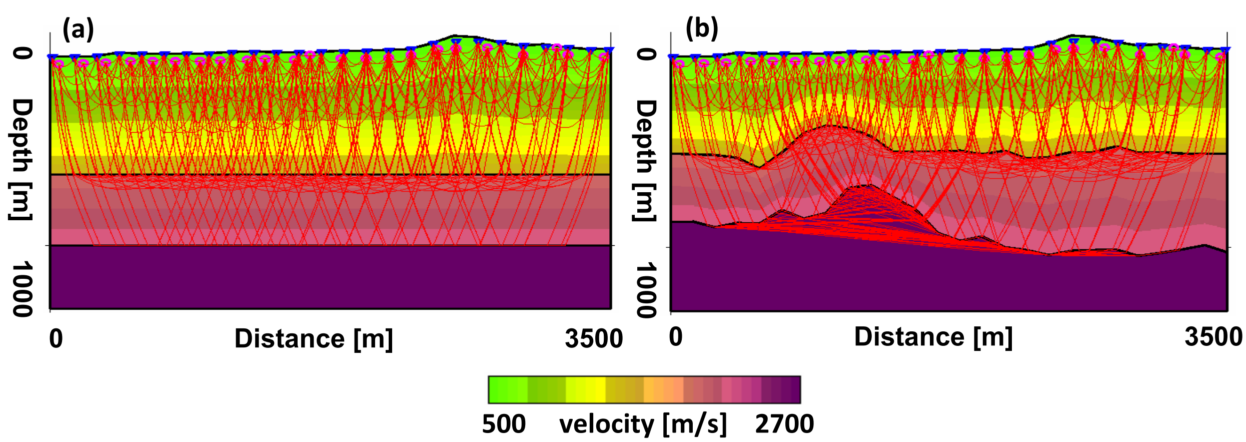

Figure 5: First arrival tomography using depth-varying velocity gradients and *a priori* information. (a) The initial model. (b) The solution model after 10 iterations.

EDITED REFERENCES

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