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# Hybrid Acoustic-elastic Modeling Method Using Adaptive Grid Finite Difference Scheme in Marine Environment

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# SUMMARY

Shear- and mode-converted waves provide rich information for further improving the image quality of complex geological structures and quantification of reservoir characterization. Multi-component seismic data acquisition has become more and more popular for surface seismic, Ocean Bottom Cable (OBC), and Vertical Seismic Profile (VSP) over the recent years. The staggered-grid finite-difference (FD) method with velocity-stress wave equation is often used for elastic wave simulation. Conventional elastic wave simulation uses fixed-grid discretization throughout the 3D volume. It requires a huge computing cost and encounters oversampling with high velocity. In order to make elastic modeling more cost effective in marine environment, we propose a new hybrid method to perform 3D elastic modeling. We implement the first-order acoustic wave equation used in water layer and the velocity-stress elastic wave equation with adaptive grid in solid sediment. The numerical result of the hybrid method matches very well with the conventional approach using fixed-grid implementation with full elastic wave equation. Test on the SEAM model also demonstrates that this method is capable for modeling surface seismic, OBC and VSP acquisition geometry with greatly improved efficiency and less used computing resource.



#### Introduction

Shear- and mode-converted waves provide rich information for further improving the image quality of complex geological structures and quantification of reservoir characterization. Elastic forward modeling is helpful in the study of interpretation problems and the optimization of novel seismic-acquisition design. The staggered-grid finite-difference (FD) method with velocity-stress elastic wave equation is often used for elastic wave simulation. Conventional FD elastic modeling is performed with full elastic wave equations using fixed-grid discretization throughout the 3D volume. However, it requires a huge computing cost and encounters oversampling issues (Fornberg 1988), especially in marine seismic acquisition owing to the slow water velocity. To improve the computation efficiency, Komatitsch et al. (1999) developed a spectral-element method to solve the acoustic-elastic coupled modeling algorithm. Lim et al. (2008) proposed a second-order acoustic-elastic wave equation scheme. Their scheme assigned material properties within each grid volume and applied them to the displacement-stress equation. To avoid spatial oversampling with high velocity, Pitarka (1999) proposed a method for discrete, spatial-differential operators in 3D staggered-grid formulation with adaptive grid spacing in pure elastic media.

In this paper, we propose a hybrid method to perform 3D elastic modeling in marine environments. It combines the first-order acoustic wave equation used in water layer and the velocity-stress elastic wave equation with adaptive grid in solid sediment. By applying the first-order acoustic/elastic perfectly matched layer (PML) boundary condition, the numerical boundary reflections are greatly absorbed. The accuracy of the wavefield with the hybrid method is compared with that of the conventional approach with fixed-grid implementation of full elastic equation. Simulations of surface seismic, OBC, and VSP acquisitions are also demonstrated.

# Method

To implement elastic wave modeling in marine environments, we apply the first-order acoustic wave equation and the velocity-stress elastic wave equation with adaptive grid scheme. The velocity model is split into water zone and solid elastic zones. Water velocity is used to calculate grid spacing in the water zone, while shear-wave velocity is used to calculate grid spacing in solid elastic zones (Figure 1). Adaptive grid spacing is determined in the different elastic zones based on certain criteria. The scheme of determining adaptive FD coefficients was proposed by Pitarka (1999).

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Figure 1: A grid layout of hybrid acoustic-elastic modeling method.

In water layer, we apply the first-order acoustic wave equation:

$$\begin{aligned} \dot{v}_i &= p_i + s_i \\ \dot{p} &= V_w^2 v_{i,i} \end{aligned}$$
 (1)

Here  $\dot{v}_i (i = x, y, z)$  stands for the first derivative over time of the *i*th particle velocity component;  $p_i$  is the first derivative over *i*th component of the pressure component;  $\dot{p}$  is the first derivative over time of the pressure component;  $V_w$  is the water velocity; and  $s_i$  is the source function. The grid spacing is calculated by  $V_w/(f_{max} * n)$ . Here,  $f_{max}$  is the maximum frequency to be modelled, *n* is the grid number per minimum wavelength.



In the solid-elastic medium, we apply the velocity-stress equations:

$$\rho \dot{v}_{i} = \tau_{ij,j}$$

$$\tau_{ij} = \lambda \delta_{ij} v_{k,k} + \mu (v_{j,i} + v_{i,j})$$

$$(2)$$

Where,  $\lambda$  and  $\mu$  are the Lamé constants,  $\dot{v}_i$  stands for the first derivative over time of the *i*th particle velocity component,  $\tau_{ij,j}$  is the first derivative over *j*th (*j*=*x*, *y*, *z*) component of the stress tensor.  $\delta_{ij}$  is the component of the Kronecker tensor, and  $v_{k,k} = v_{x,x} + v_{y,y} + v_{z,z}$ . A special treatment of the boundary condition is needed to avoid numerical side boundary reflection. Collino and Tsogka (2001) proposed an elastic PML boundary condition to absorb the boundary reflections. For the first-order acoustic wave equation, a PML boundary condition is described below:

$$p = p^{1} + p^{2} + p^{3}$$

$$(\partial_{t} + d_{x})v_{x} = \partial_{x}p, \qquad (\partial_{t} + d_{y})v_{y} = \partial_{y}p, \qquad (\partial_{t} + d_{z})v_{z} = \partial_{z}p \qquad (3)$$

$$(\partial_{t} + d_{x})p^{1} = V_{w}^{2}\partial_{x}v_{x}, \qquad (\partial_{t} + d_{y})p^{2} = V_{w}^{2}\partial_{y}v_{y}, \qquad (\partial_{t} + d_{z})p^{3} = V_{w}^{2}\partial_{z}v_{z}$$

Where, p is the pressure component, v is the particle velocity component, and  $d_i(i = x, y, z)$  is the damping function defined in Collino and Tsogka (2001). The superscripts of p represent the split PML components in x-, y-, and z- directions.

#### Examples

A simple two-layer model is introduced to verify the accuracy of this hybrid method. As shown in Figure 2a, the first layer is water, and the second layer is solid-elastic medium. The Ricker wavelet with a maximum frequency of 20 Hz is used. Figure 2b shows a snapshot of the vertical component of particle velocity at 0.7 s. Figure 2c shows the comparison of zero-offset traces of the vertical component obtained by analytical results generated by the Cagniard-De Hoop's technique (de Hoop 1960), conventional elastic modelling result, and hybrid elastic modeling with adaptive grid result. The amplitudes of three traces are scaled for comparison. The direct arrival and reflected wave match well in terms of wave shape and travel time. The memory requirement of our method is 40% less than the conventional approach. The computation is also about 57% faster than the conventional approach.



*Figure 2:* (a) Velocity model with subzones; (b) Snapshot of the vertical component of particle velocity at t=0.7 sec; (c) Comparison of the vertical component at zero-offset among analytical solution, conventional elastic modeling, and hybrid elastic modeling.



We further perform a reality test on the 3D SEG Advanced Modeling Program (SEAM) dataset with OBC and VSP acquisition geometries. It contains a major salt body and suite of reservoirs. Figure 3a shows a P-wave velocity model with a source and receivers' position. S-wave velocity is set to half of the P-wave velocity. The computation aperture is limited to a crossline aperture of 8 km and an inline aperture of 6 km with modeling a depth of 10 km. The total recording time is 10 s with a 4-ms sample interval. The Ricker wavelet is used with a maximum frequency of 28 Hz. By applying our method, the velocity model is split into multiple subzones. The first zone is a water zone and the rest zones are elastic zones. Figure 3b shows an ocean-bottom topography of the SEAM model. OBC data will help obtain more information regarding the subsurface geological layers from which the reflections or mode conversions occur. Figure 3c shows snapshots of the vertical component of the particle velocity generated by acoustic and hybrid elastic modeling at time 1.2 s. Some converted wave energy is trapped in the area between ocean bottom and top of salt. The perturbations of intra-salt sediment inclusions, or dirty salt, cause scattered wave propagation. The vertical component of elastic OBC data (Figure 3d) shows more converted wave contribution from the salt boundary. It enables the best illumination and imaging in complex structures. The hybrid elastic modeling scheme can also be used for VSP acquisition geometry (Figure 3e). It will automatically assign receivers to different subzones and record P-, S-, and converted waves simultaneously. The memory requirement in this test is 48% less than the conventional approach and saves more than 50% computing time.

### Conclusions

To make elastic modeling more cost effective in marine environments, we proposed a hybrid method to combine the acoustic and elastic wave equations with adaptive grid implementation. An acoustic wave equation is used for simulating wave propagation in water layer, while the elastic wave equation is used in the solid-elastic medium below the water bottom. With implementing the adaptive grid scheme to acoustic/elastic wave propagation, oversampling in high-velocity areas is avoided, resulting in a tremendous improvement in computing efficiency. The numerical result of the hybrid method matches very well with the conventional approach using fixed-grid implementation with full elastic wave equation. The test on the SEAM model also demonstrates that this method is capable of modeling surface seismic, OBC, and VSP acquisition geometries with greatly improved efficiency and less frequently used computing resources.

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*Figure 3:* (a) *P*-wave velocity of the original acoustic SEAM model; (b) The ocean bottom topography of the model. (c), (d) and (e) are comparisons of wavefield snapshot at t=1.2 sec, OBC data and VSP data of the vertical component between acoustic modeling (left panels) and hybrid elastic modeling (right panels).