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Seismic Elastic Wave Modeling with an Adaptive Staggered Grid in Tilted Transversely Isotropic Media

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Summary

Recent advances in seismic data processing with multi-component data have shown contributions from elastic waves in tilted transversely isotropic (TTI) media. To obtain better understanding of the elastic wave propagation in TTI media, finite difference elastic modeling is becoming valuable. However, a standard staggered grid scheme require additional interpolation between certain field variables for off-diagonal derivatives, which may reduce accuracy with significant memory allocation and considerable computation time. To overcome such issues, an adaptive Lebedev staggered grid scheme is developed for TTI elastic modeling. It reduces memory usage and computation time with stable results. The preliminary experiments demonstrate the accuracy and efficiency of this scheme.

Introduction

Seismic anisotropy exists in most sedimentary rocks. In dipping shale layers, the symmetry axis of the transverse isotropic layer will be tilted by a certain dip angle. Proper treatment on tilted transversely isotropic (TTI) elastic waves can avoid misinterpretation of anisotropic waves as artifacts and provide complementary information over isotropic velocity. It is crucial to understand and exploit elastic wave propagation in TTI media. Finite difference scheme has been considered the most popular implementation to model wave propagation in elastic media.

To perform elastic modeling in TTI media with the standard staggered grid (Virieux 1986), wavefield interpolation is necessary but reduces the simulation accuracy. Saenger et al. (2000) introduced the rotated staggered grid finite difference method for modeling elastic waves in TTI media. Lisitsa and Vishnevskiy (2010) proposed the Lebedev staggered grid to simulate TTI elastic modeling. In their method, the stress and particle velocities are divided into four sub-grid groups and each group is used for elastic modeling. Bernth and Chapman (2011) compared the rotated staggered grid and the Lebedev staggered grid based on equivalent dispersion error. They concluded that the Lebedev staggered grid is preferable for TTI elastic modeling.

Conventional finite difference elastic modeling is performed with full elastic wave equations using fixed-grid discretization throughout the 3D volume. However, it requires a significant computing cost and encounters oversampling issues (Fornberg 1988). To improve the computation efficiency and avoid spatial oversampling with high velocity, Pitarka (1999) proposed a method for spatial-differential operators in staggered grid formulation with adaptive grid spacing in pure elastic media. Jiang and Jin (2013) developed a new hybrid acoustic-elastic wave modeling method with adaptive grid implementation. Their method avoids the oversampling issue and results in a tremendous improvement in computing efficiency. This abstract proposes an adaptive Lebedev staggered grid to significantly reduce the memory requirement and improve the computation efficiency for 3D TTI elastic modeling.

Methodology

In elastic medium, the velocity-stress equations are applied as:

$$\rho \dot{v}_i = \tau_{ij,j}$$

$$\dot{\tau}_{ij} = \lambda \delta_{ij} v_{k,k} + \mu \left(v_{j,i} + v_{i,j} \right)$$

Where, λ and μ are the Lame constants, $\dot{v}_i$ is the first derivative over time of the *i*th particle velocity component, $\tau_{ij,j}$ is the first derivative over *j*th (j=x, y, z) component of the stress tensor, δ_{ij} is the component of the Kronecker tensor, and $v_{k,k} = v_{x,x} + v_{y,y} + v_{z,z}$. Lisitsa and Vishnevskiy (2010) developed a Lebedev staggered grid to simulate TTI elastic modeling with high accuracy. The stress and particle velocity components are divided into four sub-grid groups (Figure 1) and stored in staggered locations.

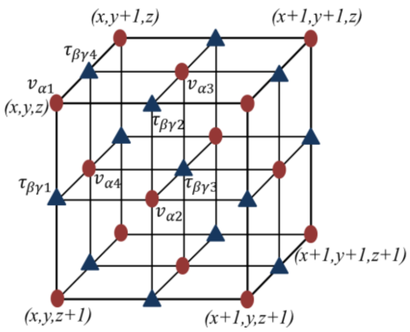

***Figure 1**: The Lebedev staggered grid. The red circle represents particle velocity component $v_\alpha(\alpha=x,y,z)$, and the blue triangle stands for stress component $\tau_{\beta\gamma}(\beta,\gamma=x,y,z)$.*

This approach removes the rotation of a gradient or the divergence of a rotation. It does not need interpolation of spatial derivatives and can provide highly accurate results. The final result will be summed by all components at four sub-grid groups into one output grid group. To reduce memory usage and save computational time, an efficient scheme is to use an adaptive grid that applies to different velocity zones (Figure 2). The interface between different zones is accomplished by the linear interpolation of field variables only in the areas of discontinuity grids or overlap zone (Figure 3). In this overlap zone, vertical derivatives are calculated by finite difference with least-square variable coefficients. For each component of particle velocities ($v_{\alpha\omega}$ ($\alpha=x,y,z$ and $\omega=1,2,3,4$) or stress $\tau_{\beta\gamma\omega}$($\beta,\gamma=x,y,z$ and $\omega=1,2,3,4$), the vertical derivatives with variable coefficients are implemented to all four sub-grid groups. In this case, each parameter set is accomplished by adaptive staggered grid scheme, and it is possible to adapt the grid spacing to velocity structure in accordance with the elastic finite difference requirement.

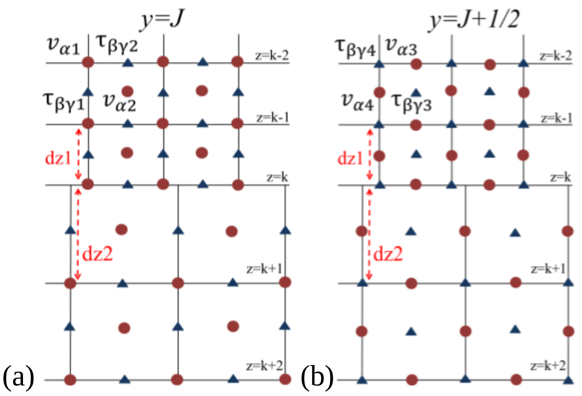

Figure 2: Adaptive Lebedev staggered grid in (a) integer grid point (y=J) and (b) in half-grid point (y=J+1/2). Here, $\alpha,\beta,\gamma = x,y,z$.

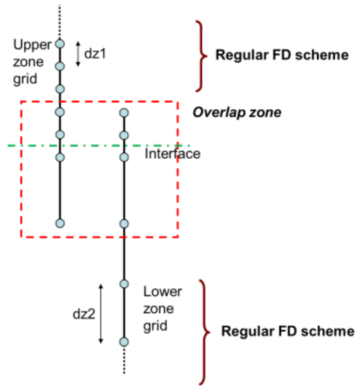

***Figure 3**: Layout of vertically variable grid with overlap zone.*

High-order finite difference scheme is implemented to avoid grid dispersion, particularly when low shear wave velocity exists. The regular finite difference with uniform grid spacing will dominate the wave propagation. However, in the overlap zone (Figure 3), a group of variable 16th-order finite difference coefficients will be applied to calculate the derivatives of each field variable. These coefficients are calculated by Taylor's expansion, which is used to approximate the exponentials of spatial derivatives of each variable (Pitarka 1999). Once the velocity model is split into different zones and grid spacing is determined in each zone from minimum S-wave velocity and maximum frequency, variable coefficients in each zone can be calculated before the modeling step. A special treatment of the boundary condition is necessary to avoid numerical boundary reflection. Here, the unsplit convolutional-PML (C-PML) (Martin and Komatitsch 2009) boundary condition is extended to incorporate with adaptive Lebedev staggered grid in TTI media. Because C-PML does not require splitting equations into separate equations, this reconfiguration is straightforward.

Numerical tests show that, for high-order finite difference scheme with variable grid, the Lebedev staggered grid requires less grids per wavelength than standard staggered grid to achieve the same dispersion error. The dispersion relation is derived as $dz = V_{min}/(Freq_{max}*N)$, where V_{min} is minimum velocity in each zone, $Freq_{max}$ is maximum frequency of modeling, and N is grid point per minimum wavelength. Spatial sampling dz is variable in different velocity zones. The stability condition of the staggered grid scheme is tested with a series of numerical tests and is satisfied by $dt=dz_{min}/(\sum|a_i|*\sqrt{dim}*V_{max})$, where dz_{min} is minimum grid spacing in each zone, $\sum|a_i|$ is the sum of absolute value of finite difference coefficients, dim is the model dimension, dim=2 if 2D modeling or dim=3 if 3D modeling, V_{max} is maximum velocity value in each zone. Time sampling rate is calculated in each velocity zone, and only the minimum sampling rate among all zones will be selected as the final propagation time sampling rate.

Numerical Examples

Figure 4 shows a displacement comparison between the analytical and numerical result of the Lebedev staggered grid. Source wavelet is a triangle wavelet with 20-Hz maximum frequency. Source excitation type is moment tensor source [0, 1, 0; 1, 0, 0; 0, 0, 0]. By comparing the relative amplitude and traveltime, the Lebedev grid implementation results match very well with the analytical results.

Furthermore, a comparison in a three-layer model between VTI and TTI media is performed. Table 1 shows the model parameters in each layer. The computation aperture is limited to a 4-km crossline aperture and 4-km inline aperture with 5-km modeling depth. The Ricker wavelet is used with a 25-Hz maximum frequency and source depth is 500 m below surface. In this example, a 60° constant tilted dip angle and 30° azimuth angle are used in the whole model. By applying an adaptive staggered grid scheme, the velocity model is split into three zones. Figure 5 shows snapshots in VTI and TTI media at 1 second. In this example, the standard adaptive staggered grid is used for VTI elastic modeling, while the adaptive Lebedev staggered grid is used for TTI elastic modeling.

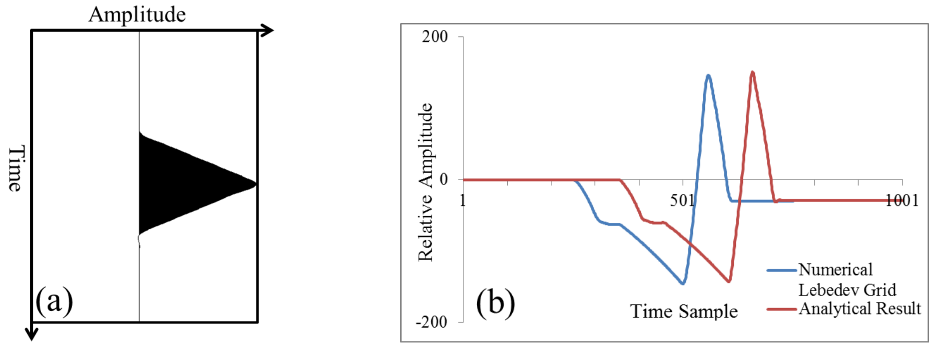

Figure 4: *(a) Triangle source wavelet; (b) displacement comparison between analytical (red line) and numerical Lebedev grid (blue line) results.*

Table 1: Anisotropic velocity parameters

	Vp (km/s)	Vs (km/s)	ε	δ	γ	Tilted dip θ	Azimuth φ
1	2.0	1.0	0.3	0.05	0.3	60°	30°
2	3.0	1.8	0.2	0.1	0.2		
3	4.0	2.4	0.15	0.08	0.15		

Table 2: Comparisons of memory usage and computation time

	VTI (Standard grid)		TTI (Lebedev grid)	
	Fixed grid	Adaptive grid	Fixed grid	Adaptive grid
Memory usage (GB)	1	0.44	1	0.34
Computation time (hr)	1	0.29	1	0.35

Table 2 compares memory usage and computation time for VTI and TTI elastic modeling with and without adaptive grid scheme. During this test, the adaptive Lebedev grid scheme saves more than 60% memory usage than the conventional fixed Lebedev grid scheme. The total computation time also decreases to 60% less than the conventional fixed Lebedev grid scheme.

Shear wave triplication, or birefringence, is clearly shown in Figure 5. The existence of the triplication depends on the strength of three anisotropic parameters (ε, δ, and γ) and the variation of tilted dip and azimuth angle. This can be observed at long offset converted wave data, long offset vertical seismic profiles (VSPs), and cross-well seismic data. With implementation of rotated symmetry axis, the polarization of S-wave will lie in different planes, which generate two different S-wave velocities, or shear-wave splitting. Shear-wave splitting helps measure the degree of anisotropy and leads to a better understanding of sediment density and crack orientation.

Conclusions

This abstract proposed an adaptive Lebedev staggered grid scheme for TTI elastic wave modeling. Lebedev staggered grid avoids extra interpolation of off-diagonal derivative that exists in the standard staggered grid scheme. Combining Lebedev staggered grid with adaptive grid scheme, oversampling in high-velocity areas is avoided. This method provides the capability to model large datasets with less memory requirement and faster computational time compared to the traditional TTI elastic modeling scheme. Implementing TTI elastic modeling will help seismic processors and interpreters understand shear-wave splitting, or the degree of anisotropy, which leads to a better understanding of the sediment property and detecting the fracture orientation.

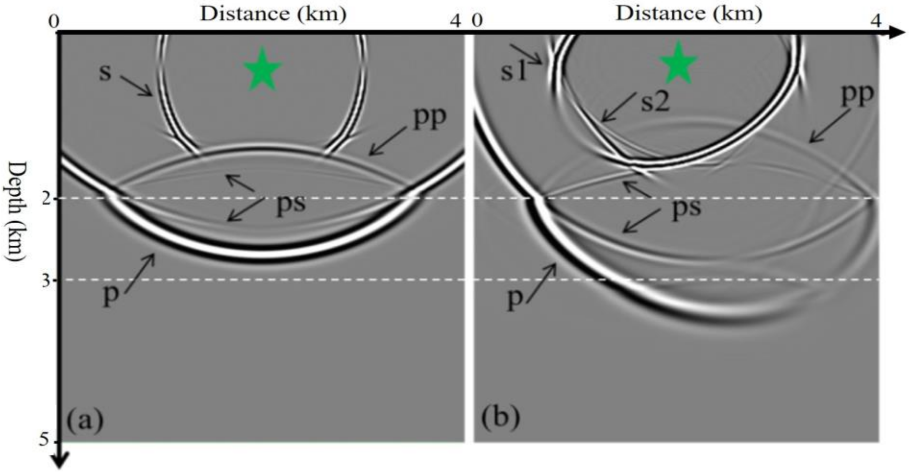

Figure 5*: Snapshots of vertical component in (a) VTI and (b) TTI media. Here, t = 1 second. The green star is the source position. In TTI media, tilted angle and azimuth angle rotate the wavefield by a certain angle, which results in clear shear-wave splitting shown in (b).*

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